

谐波齿轮传动 S 型齿形设计与啮合性能研究

Research on S-Shaped Tooth Profile Design and Meshing Performance of Harmonic Gear Drive

摘 要

谐波减速器作为机器人关节驱动的核心零部件,具有大减速比、结构紧凑、背隙小等显著优势,在高动态响应精密控制和大载荷动力传输等应用场景中展现出优异的服役性能,直接决定了机器人的运动精度、动态响应能力和使用寿命。然而,随着应用场景向高速、重载、长寿命方向发展,谐波减速器系统化齿形主动设计理论缺失问题逐渐暴露。传统的刚轮与柔轮齿廓设计方法尚未形成与模数关联设计的统一齿廓参数化建模体系,且初始齿廓参数的可行域范围缺乏严格的数学定义。此外,由于柔轮非线性变形机理复杂且难以建立精确解析模型,导致对柔轮与刚轮齿面啮合-载荷协调匹配机制的研究不足,使得负载及安装误差对齿面接触特性的影响规律尚未明晰,成为制约谐波减速器性能提升的瓶颈。为突破以上存在的技术瓶颈,本文开展谐波减速器运动学建模、齿形设计及参数优化、齿面承载接触特性分析等研究;建立了基于支撑函数设计柔轮中性层曲线的谐波减速器运动学模型,提出了具有连续二次共轭特性的谐波减速器高柔性 S 型齿形设计方法,揭示了齿面修形对谐波传动齿面接触特性的影响规律。研究成果对谐波减速器啮合机理揭示、齿形标准化设计以及啮合性能控制具有重要意义,为形成谐波减速器系统化齿形设计流程提供了理论支撑。

论文的主要内容如下:

(1) 构建了基于支撑函数柔轮中性层曲线设计的谐波减速器运动学模型,阐明了柔轮齿运动轨迹与中性层曲线的数学关系,利用瞬心概念对谐波减速器运动学模型进一步阐述,推导谐波传动瞬时传动比、滑移率及综合曲率半径计算公式;基于多坐标系下瞬心线分析,推导了基于 Willis 定理的谐波减速器刚柔轮齿廓共轭啮合方程,建立了刚柔轮齿廓参数选取的可行域范围与瞬心线的几何约束边界,揭示了啮合界限点与谐波减速器连续二次共轭的几何关系。

(2) 基于滚刀法向截面 S 齿廓方程定义,将齿廓设计参考点控制为啮合界限点,构建了从滚刀→柔轮齿→刚轮齿的系统化齿形设计方法,研究齿廓设计参数对重合度、滑移率及综合曲率半径的影响规律;提出了谐波减速器多齿啮合变形协调及静力平衡计算方法,突破以重合度作为单一目标优化的局限性,综合考虑齿面滑移率、啮合点综合曲率半径及重合度,形成基于齿面接触压力最小的 S 齿形谐波减速器无侧隙齿形设计及优化方法。

(3) 基于滚齿加工空间几何关系及双包络啮合理论,提出了考虑综合修形的柔轮齿面精密创成方法,揭示齿向鼓形修整及跨棒距变化对柔轮齿面几何形貌的影响规律;提出了考虑刚轮齿廓修形的锥形车齿刀设计方法,基于车齿加工原理,

研究跨棒距变化对刚轮齿面几何形貌的影响规律；形成考虑综合修形的 S 齿形谐波减速器刚柔轮齿面精确建模方法。

（4）建立了 S 齿形谐波减速器齿面承载接触分析模型，研究刚轮齿廓抛物线修形对刚柔轮齿面二次共轭及两点共轭特性的影响；建立了考虑柔轮齿向修形的谐波减速器空间啮合侧隙计算方法，研究啮合侧隙分布对齿面接触状态与应力分布均匀性的影响规律，形成基于啮合侧隙控制的 S 齿形谐波减速器啮合特性优化方法。

（5）开展了 S 齿形谐波减速器样机研制及试验研究，分别研制加工双刚轮谐波减速器及礼帽形谐波减速器，通过双刚轮谐波减速器齿面磨痕实验验证刚柔轮齿面二次共轭特性的存在，通过谐波减速器综合性能试验台及摆臂式振动试验台对比分析不同修形量下礼帽形谐波减速器传动误差、刚性、启动扭矩、传动效率及振动特性等性能指标，验证本文齿形设计及修形方法在工程应用的可行性。

关键词：谐波减速器；S 齿形；连续二次共轭；齿面修形；齿面接触特性

Abstract

As a core component of robotic joint drives, harmonic reducers exhibit significant advantages such as high reduction ratios, compact structure, and minimal backlash. They demonstrate outstanding performance in applications requiring high-dynamic-response precision control and heavy-load power transmission, with their performance directly determining a robot's motion accuracy, dynamic response capability, and service life. However, as applications evolve toward high-speed, heavy-load, and long-life scenarios, the lack of systematic active design theory for tooth profiles in harmonic reducers has become increasingly apparent. Traditional design methods for the tooth profiles of the circular spline and flexspline have yet to establish a unified parametric modeling system for tooth profile parameters correlated with the module, and the feasible range of initial tooth profile parameters lacks a rigorous mathematical definition. Furthermore, due to the complex nonlinear deformation mechanism of the flexspline and the difficulty in establishing an accurate analytical model, research on the meshing-load coordination mechanism between the flexspline and circular spline remains insufficient. As a result, the influence of load and installation errors on tooth contact characteristics has not been fully clarified, becoming a bottleneck restricting the performance improvement of harmonic reducers. To overcome these technical challenges, this study investigates the kinematic modeling, tooth profile design, parameter optimization, and tooth contact characteristics of harmonic reducers. A kinematic model of harmonic reducers based on the support function design of the neutral curve of the flexspline is established. A high-flexibility S-shaped tooth profile design method with continuous double-conjugate characteristics for harmonic reducers is proposed, and the influence of tooth profile modification on the contact characteristics of harmonic transmission is revealed. The research findings hold significant importance for understanding the meshing mechanism, tooth profile design, and meshing performance control of harmonic reducers, providing theoretical support for establishing a systematic tooth profile design process for harmonic reducers.

The main contents of this dissertation are as follows:

(1) A kinematic model of the harmonic reducer was established based on the support function design of the flexspline neutral curve, clarifying the mathematical relationship between the tooth motion trajectory of the flexspline and the neutral curve. Using the

concept of instant centers, the kinematic model of the harmonic reducer was further elaborated, and formulas for the instantaneous transmission ratio, sliding rate, and comprehensive curvature radius of harmonic drive were derived. Based on the analysis of instant center lines in multiple coordinate systems, the conjugate meshing equation of the circular spline and flexspline tooth profiles was derived according to Willis' theorem. The feasible domain for selecting tooth profile parameters of the circular spline and flexspline was established, along with the geometric constraint boundaries of the instant center lines. The geometric relationship between the meshing boundary point and the continuous double-conjugate characteristics of the harmonic reducer was revealed.

(2) Based on the definition of the S-shaped tooth profile equation in the normal section of the hob, the tooth profile design reference point was controlled as the meshing boundary point, establishing a systematic tooth profile design methodology from the hob $\rightarrow$ flexspline tooth $\rightarrow$ circular spline tooth. The influence of tooth profile design parameters on the contact ratio, sliding rate, and comprehensive curvature radius was investigated. A multi-tooth meshing deformation coordination and static equilibrium calculation method for harmonic reducers was proposed, overcoming the limitation of using contact ratio as the sole optimization objective. By comprehensively considering the tooth surface sliding rate, meshing point comprehensive curvature radius, and contact ratio, an S-shaped tooth profile design and optimization method for backlash-free harmonic reducers was developed, aiming to minimize tooth contact pressure.

(3) Based on the spatial geometric relationship of gear hobbing and the dual-envelope meshing theory, a precision generation method for the flexspline tooth surface incorporating comprehensive modifications was proposed, revealing the influence of tooth crowning and over ball diameter (OBD) variation on the geometric morphology of the flexspline tooth surface. A conical skiving cutter design method considering circular spline tooth profile modification was introduced. By analyzing the skiving machining principle, the impact of OBD variation on the geometric morphology of the circular spline tooth surface was investigated. An accurate modeling approach for the tooth surfaces of both the circular spline and flexspline in S-shaped harmonic reducers was established, incorporating comprehensive modification considerations.

(4) An S-shaped tooth profile load-bearing contact analysis model for harmonic reducers was established, and investigate the influence of parabolic modification of the circular spline tooth profile on the double-conjugate and two-point conjugate characteristics of the meshing tooth surfaces. A spatial meshing backlash calculation

method for harmonic reducers incorporating flexspline tooth crowning modification was developed. This method examines the effects of backlash distribution and assembly errors on tooth contact patterns and stress distribution uniformity, This led to the development of a meshing performance optimization methodology for S-profile harmonic drives based on precise clearance control.

(5) Prototype development and experimental analysis for S-shaped harmonic reducers were conducted. Dual-circular-spline and top-hat harmonic reducers were manufactured. The existence of double-conjugate characteristics between the flexspline and circular spline tooth surfaces was verified through tooth surface wear pattern tests on the dual-circular-spline harmonic reducer. Using a comprehensive performance test bench and a pendulum-type vibration test platform, Comprehensive performance tests on dedicated test benches and swing-arm vibration platforms comparatively analyzed transmission error, rigidity, starting torque, efficiency, and vibration characteristics under different modification parameters. These experiments validated the engineering feasibility of the tooth profile design and modification methods proposed in this study.

Keywords: Harmonic drive; S-shaped tooth profile; Continuous double conjugation; Tooth surface modification; Analysis of meshing characteristics